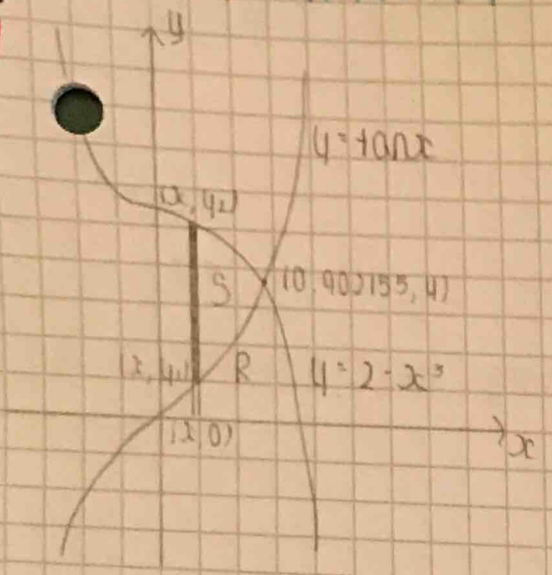
2001: PART A

1)



a) Graph cuts x axis:

$2 - x^3 = 0$

$x^3 = 2$

$x = 2^{1/3}$

2 curves intersect:

$2 - x^3 = \tan x$

$x = 0.902155$ using G.D.C.

$\text{Area } R = \int_0^{0.902155} \tan x \, dx + \int_{0.902155}^{2^{1/3}} (2 - x^3)\, dx$

$= 0.729$ to 3.d.p.

b) $\text{Area } S = \int_0^{0.902155} (2 - x^3) - \tan x \, dx$

$= 1.161$ to 3.d.p.

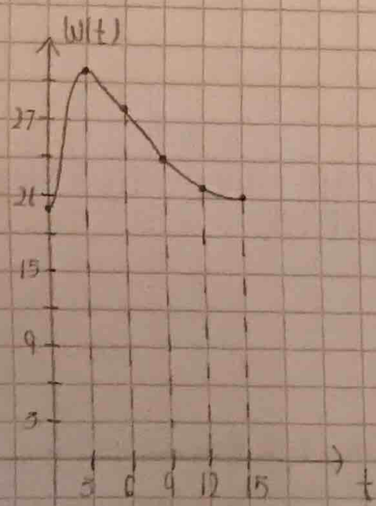
c) $R(x) = y_2 - 0 = 2 - x^3$

$r(x) = y_1 - 0 = \tan x$

$\text{Volume} = \pi \int_0^{0.902155} (2 - x^3)^2 - (\tan x)^2 \, dx$

$= 8.352$ to 3.d.p.

2)



a) $W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{21 - 24}{6} = -\frac{3}{6} = -0.5$ °C/day

b) $\text{Area} = \frac{15 - 0}{2(5)} \left[W(0) + 2(W(3) + W(6) + W(9) + W(12)) + W(15) \right]$

$= \frac{15}{10}\left(20 + 2(31 + 28 + 24 + 22) + 21\right)$

$= \frac{3}{2}(251)$

$= 376.5$

$\int_0^{15} W\, dt = 376.5 \therefore W_{AV} = \frac{1}{15 - 0}\int_0^{15} W\, dt = \frac{1}{15}(376.5) = 25.1$ °C

c) $P(t) = 20 + 10te^{(1-t/3)}$

$P'(t) = 10e^{(1-t/3)} - \frac{10}{3}te^{(1-t/3)}$

$P'(12) = 10e^{-4} - \frac{10(12)}{3}e^{-4}$

$= 10e^{-4} - 40e^{-4}$

$= 10e^{-4}(1-4)$

$= -30e^{-4}$

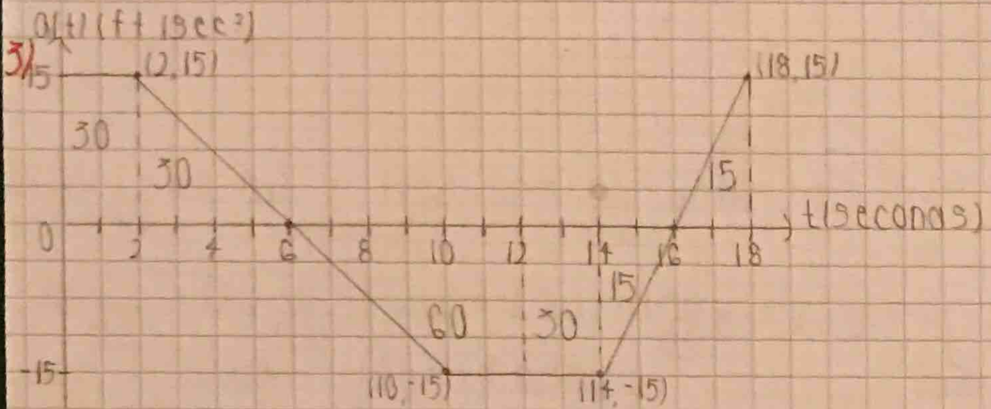
$= -0.549$ to 3 d.p. °C/day

This means that the temperature is decreasing at a rate of 0.549 °C/day when $t = 12$ days

d) $P(t)_{AV} = \frac{1}{15-0}\int_0^{15} P(t)\,dt = \frac{1}{15}\int_0^{15} (20 + 10te^{(1-t/3)})\,dt$

$= 25.757$ °C

3)

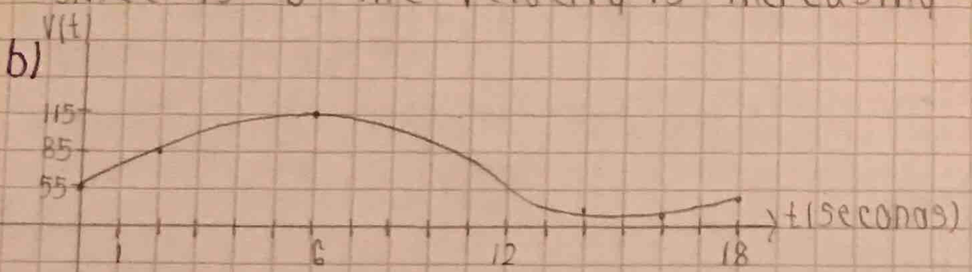


a) $v'(t) = a(t)$

$a(2) = 15 \therefore v'(t) = 15$

Since $15 > 0$ the velocity is increasing at $t = 2$

b)



$\int_0^{12} a(t)\,dt = v(12) - v(0) = 0$

c) $v(0) = 55$

Rel. max at $t = 6$ since $v'(t)$ changes sign from +ve to −ve
Absolute max occurs at rel. max or endpoints

$v(6) = 55 + \int_0^6 a(t)\,dt = 115 > v(0)$

$\int_6^{18} a(t)\,dt < 0$ so $v(18) < v(6)$

The absolute max velocity is 115 ft/sec at $t = 6$

The cars velocity is never 0

the absolute min occurs at $t = 16$ where $v(t) = 115 + \int_{6}^{16}$

$= 115 - 105 = 10 > 0$

2

001: PART B Form B

$h'(x) = \dfrac{x^2-2}{x}, \; x \neq 0, \; h(4) = -3$

a) $h'(x) = 0$

$\dfrac{x^2-2}{x} = 0$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$h'(x)$: $-$ $+$ $-$ $+$ at $-\sqrt{2}$, 0, $\sqrt{2}$

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There is a local minimum at $x = \pm\sqrt{2}$ since $h'(x)$ changes sign from -ve to +ve at $x = -\sqrt{2}$ and $x = \sqrt{2}$

b) $h''(x) = \dfrac{x(2x) - (x^2-2)(1)}{x^2} = \dfrac{2x^2 - x^2 + 2}{x^2} = \dfrac{x^2+2}{x^2}$

$h''(x) = 0$

$\dfrac{x^2+2}{x^2} = 0$

$x^2 + 2 = 0$

$x^2 \neq -2$

$h''(x)$: $+$ $+$ at 0

$\therefore h(x)$ is concave up for $(-\infty, \infty)$, $x \neq 0$

c) Equation of tangent: $(4, -3)$

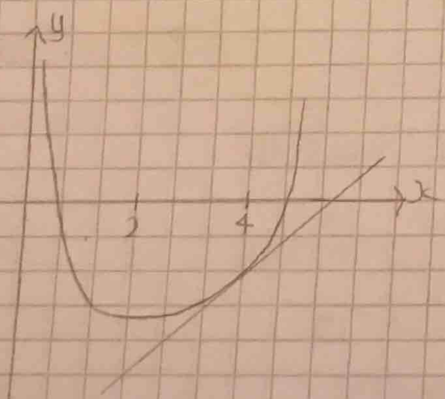
$h'(4) = \dfrac{16-2}{4} = \dfrac{14}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x - 4)$

$2y + 6 = 7x - 28$

$7x - 2y = 34$

d)



Lies below the graph of h since the graph of h is concave up for $x > 4$

$f(x) = 4x^3 + ax^2 + bx + k$

a) $f'(x) = 12x^2 + 2ax + b$
$12x^2 + 2ax + b = 0$

$f''(x) = 24x + 2a$
$24x + 2a = 0$

$f'(-1) = 12 - 2a + b = 0$
$12 - 2(24) + b = 0$
$12 - 48 + b = 0$
$b = 36$

$f''(-2) = -48 + 2a = 0$
$2a = 48$
$a = 24$

$a = 24, b = 36$

b) $\int_0^1 f(x)\,dx = 32$
$\int_0^1 (4x^3 + 24x^2 + 36x + k)\,dx = 32$
$\left[x^4 + 8x^3 + 18x^2 + kx\right]_0^1 = 32$
$(1 + 8 + 18 + k) - 0 = 32$
$k + 27 = 32$
$k = 5$

6) $\left(3, \frac{1}{4}\right)$ is on $y = f(x)$ and $\frac{dy}{dx} = y^2(6 - 2x) = 6y^2 - 2xy^2$

a) $\frac{d^2y}{dx^2} = 12y\frac{dy}{dx} - \left[2y^2 + 2x(2y)\frac{dy}{dx}\right]$
$= 12y\frac{dy}{dx} - \left(2y^2 + 4xy\frac{dy}{dx}\right)$
$= 12y\frac{dy}{dx} - 2y^2 - 4xy\frac{dy}{dx}$
$= \frac{dy}{dx}(12y - 4xy) - 2y^2$
$= 4y(3 - x)\frac{dy}{dx} - 2y^2 = 4y(3 - x)(y^2(6 - 2x)) - 2y^2 = 4y^3(3 - x)(6 - 2x) - 2y^2$
$= 8y^3(3 - x)^2 - 2y^2$

at $\left(3, \frac{1}{4}\right)$ $\frac{d^2y}{dx^2} = 8\left(\frac{1}{4}\right)^3(3 - 3)^2 - 2(3^2)$
$= -2\left(\frac{1}{4}\right)^2$
$= -\frac{2}{16}$
$= -\frac{1}{8}$

b) $\frac{dy}{dx} = y^2(6 - 2x)$
$\int \frac{dy}{y^2} = \int (6 - 2x)\,dx$
$-\frac{1}{y} = 6x - x^2 + c$
$f(3) = 1/4$
$-\frac{1}{1/4} = 6(3) - 3^2 + c$
$-4 = 18 - 9 + c$
$-4 = 9 + c \Rightarrow c = -13$

$-\frac{1}{y} = 6x - x^2 - 13$
$y = -\frac{1}{6x - x^2 - 13}$
$y = \frac{1}{x^2 - 6x + 13}$